The Economics of Competition under Network Externalities

Presenter: Shugang Hao

Pillar of Engineering Systems and Design
Singapore University of Technology and Design (SUTD)

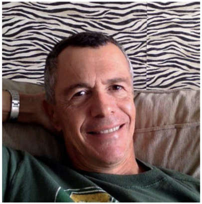
September 14, 2023

About SUTD

- A new public university established in 2009.
- Was established in collaboration with MIT.
- Ranking in the world: 21st in Telecommunication Engineering according to ShanghaiRanking 2023.

Acknowledgement

Prof. Lingjie Duan



Prof. Costas Courcoubetis

Results here are available at

- **S. Hao** and L. Duan, "To Help or Disturb: Introduction of Crowdsourced WiFi to 5G Networks," IEEE Transactions on Mobile Computing (**IEEE TMC**), vol. 22, no. 9, pp. 5583-5596, 1 Sept. 2023.
- **S. Hao** and L. Duan, "Regulating Competition in Age of Information under Network Externalities," IEEE Journal on Selected Areas in Communications (**IEEE JSAC**), vol. 38, no. 4, pp. 697-710, 2020.
- **S. Hao** and L. Duan, "Economics of Age of Information Management under Network Externalities," in **ACM MobiHoc** Symposium, Catania, Italy, July 2-5, 2019.

Background & Motivation

Background: Escalating Demands on Real-time Information

Internet users are less patient to bear any outdated information.


Forbes

FORBES > SMALL BUSINESS

Meeting The Demand For Real-Time Digital User Experiences

Sunil Thomas Former Forbes Councils Member
Forbes Business Council
COUNCIL POST | Membership (Fee-Based)

Jul 16, 2021, 07:30am EDT

- Quality of Service (QoS) should keep improving/promoting.

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- Quality of Service (QoS) should keep improving/promoting.
- Platforms need to provide fresh information update.

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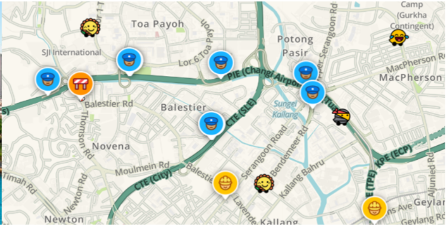
Background: Crowdsourcing

Crowdsourcing:

- platforms invite crowd to provide resource/information update,
- and provide enhanced service to all the users.

Crowdsourced WiFi Fon's Network at Australia



Waze Live Map on Real-Time Traffic Information

Users incur positive network externality with each other.

Background: Competition in Crowdsourcing

Competition arises between crowdsourcing and traditional platforms.

- E.g., Telenor vs OpenSpark for mobile users.

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Competition also arises between crowdsourcing platforms.

- E.g., Waze vs Gasbuddy for content delivery network.

Research Orientation

In this talk, we will investigate

- impact of crowdsourcing WiFi on existing 5G networks: Fon vs BT.

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Negative and positive network externalities incur among different entities.

Part I:
To Help or Disturb: Introduction of Crowdsourced WiFi to 5G Networks

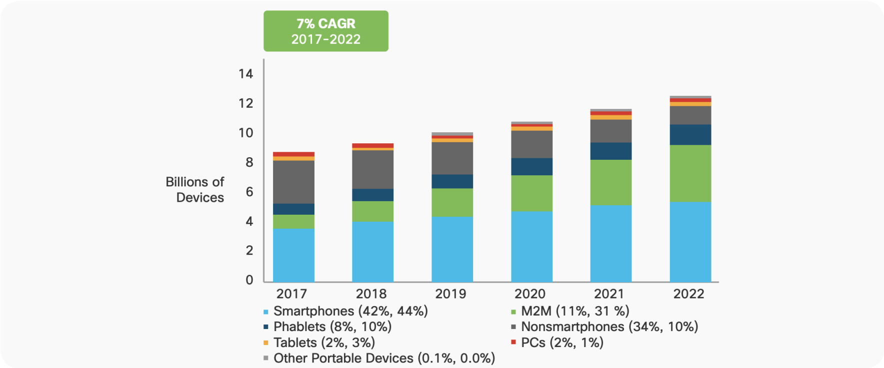
Part II:
Regulating Competition in Age of Information under Network Externalities

Overview

1. Part I: To Help or Disturb: Introduction of Crowdsourced WiFi to 5G Networks
   - Background: WiFi's Complementarity for 5G Networks
   - System Model
   - Equilibrium Analysis
   - Summary

Background: Overwhelming Mobile Devices

Figure 4. Global Mobile Devices and Connections Growth



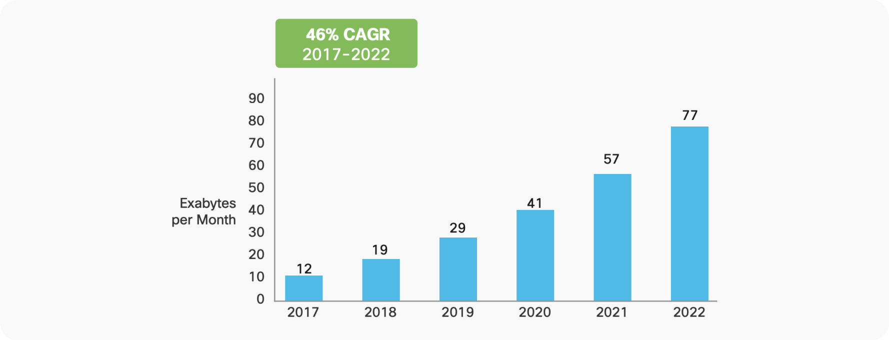
Note: Figures in parentheses refer to 2017, 2022 device share.
Source: Cisco VNI Mobile, 2019.

Rapid proliferation of mobile devices.

Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2017–2022, 2019, [online] http://media.mediapost.com/uploads/CiscoForecast.pdf

Background: Overwhelming Mobile Data Traffic

Figure 2. Cisco Forecasts 77 Exabytes per Month of Mobile Data Traffic by 2022



Source: Cisco VNI Mobile, 2019

Mobile data traffic has been ever-increasing overwhelmingly with rapid proliferation of mobile devices.

Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2017–2022, 2019, [online] http://media.mediapost.com/uploads/CiscoForecast.pdf

Background: Profile Wireless Technology Issues

To meet overwhelming data traffic demand,

[1] GSMA, "Estimating the mid-band spectrum needs in the 2025-2030 time frame," Jul. 2021, [online] Available: https://www.gsma.com/spectrum/wp-content/uploads/2021/07/Estimating-Mid-Band-Spectrum-Needs.pdf.

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  - An individual AP has small service coverage.
  - It is difficult and costly to deploy a ubiquitous WiFi network.

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Background: Crowdsourced WiFi Community

Crowdsourced WiFi community network has emerged to

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Fon's crowdsourced WiFi network

- has included over 23 million APs,

BT Fon WiFi Map, https://www.btwifi.co.uk/find/.

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Fon's crowdsourced WiFi network

- has included over 23 million APs,
- and is fast expanding to cover many populous and crowded places.

WiFi hotspots by Fon in London area.

BT Fon WiFi Map, https://www.btwifi.co.uk/find/.

Background: WiFi's Complementary for 5G

The crowdsourced WiFi community network's coverage

- is still not comparable to the ubiquitous 5G coverage,

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Co-existing 5G and crowdsourced WiFi networks:

+ British Telecom and Fon in the United Kingdom,
- Telenor and OpenSpark in Finland.

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Features of 5G and crowdsourced WiFi networks:

Features	5G	Crowdsourced WiFi
Coverage	Ubiquitous	Limited
Network Externality	Negative	Positive & Negative

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WiFi's complementarity for 5G:

5G users may add on the crowdsourced WiFi.

Research Questions

We will answer, after the introduction of the crowdsourced WiFi network,

how will 5G users add on the crowdsourced WiFi network?

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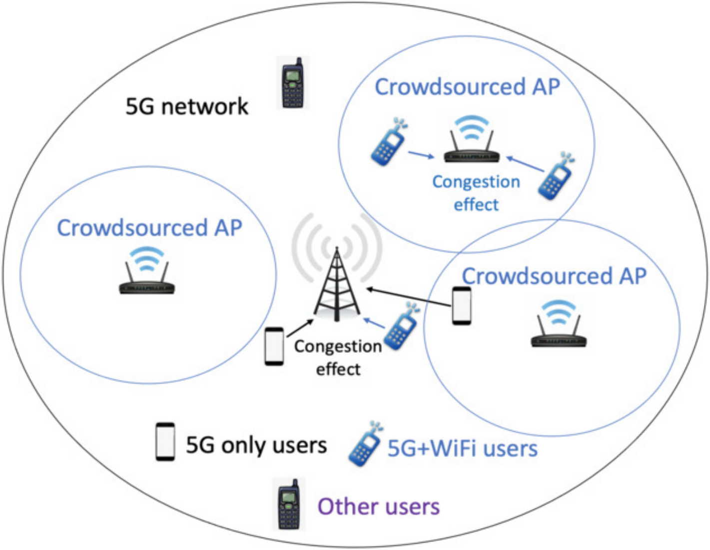
how will users' payoffs change?

System Model



Co-existence of the 5G and crowdsourced WiFi networks.

System Model before the Introduction of the Crowdsourced WiFi

We consider N users in total as potential subscribers.

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5G operator's profit:

$$\bar{\pi}_1(\bar{p}_1) = N \cdot \bar{x}_1(\bar{p}_1) \cdot \bar{p}_1,$$

- $\bar{x}_1 \in [0, 1]$: the user fraction of 5G subscription,
- $\bar{p}_1$: price (e.g., annual or monthly).

System Model Before the Introduction of the crowdsourced WiFi (Cont.)

We consider N users in total as potential subscribers.

The user's payoff of 5G subscription is given by:

$$\bar{u}_1(\theta) = V_1 - \frac{N\bar{x}_1}{Q}\theta - \bar{p}_1,$$

- V_1: positive value (e.g., for mobile Internet access),

Gibbens et al., "Internet service classes under competition," IEEE Journal on Selected Areas in Communications, vol. 18, no. 12, pp. 2490-2498, Dec. 2000.

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- Q: limited 5G network capacity,
- $\frac{N\bar{x}_1}{Q}\theta$: 5G congestion cost, large with huge participation or small capacity.

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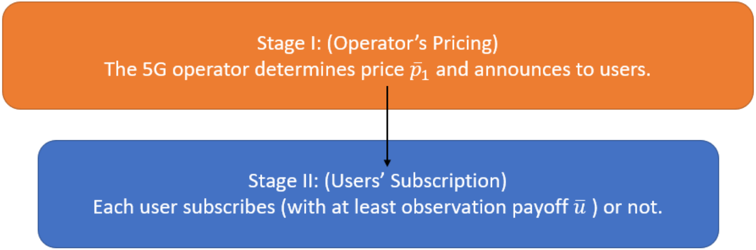
Two-stage Stackelberg Game

In practice, the network operators have more power to lead as compared to the users as followers.



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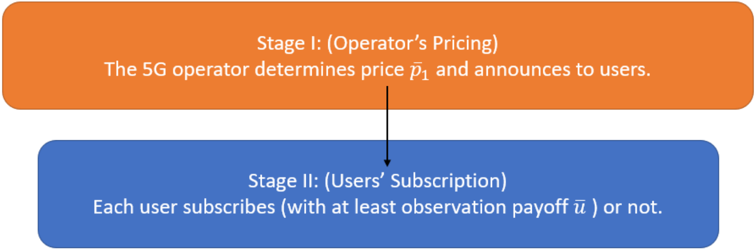
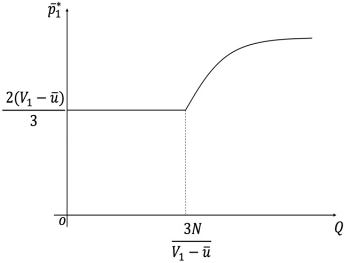


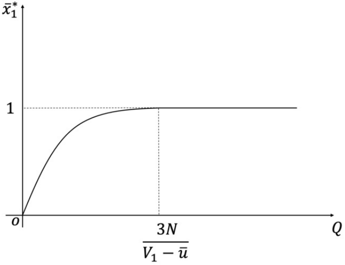
We expect all users' subscription with non-small capacity.

We derive equilibrium with backward induction.

Equilibrium of Two-stage Stackelberg Game



(a) 5G price equilibrium $\bar{p}_1^*$ versus the 5G network capacity Q



(b) Users' subscription $\bar{x}_1^*$ versus capacity Q

- The 5G operator can only charge a small price when capacity is low.
- Having non-small Q, his price increases.

System Model after the Introduction of the Crowdsourced WiFi

After the introduction of the crowdsourced WiFi,

- 5G users can further add on the crowdsourced WiFi service.

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After the introduction of the crowdsourced WiFi,

- 5G users can further add on the crowdsourced WiFi service.
- $x_1 \in [0,1]$: the fraction of N users joining the 5G network only,
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Each 5G+WiFi user

- contributes a normalized positive addition $\alpha \in (0, 1)$ to the network coverage (using his home AP),
- the overall coverage of the crowdsourced WiFi network is αx_2.

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A 5G-only User's Payoff after the Introduction of the Crowdsourced WiFi

A 5G-only user

- pays price p_1 to join the network,

A 5G-only User's Payoff after the Introduction of the Crowdsourced WiFi

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The user's payoff of 5G subscription only is

$$u_1(\theta) = V_1 - \frac{N(x_1 + x_2(1 - \alpha x_2))}{Q}\theta - p_1.$$

A 5G+WiFi User's Payoff after the Introduction of the Crowdsourced WiFi

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The user's payoff of 5G+WiFi subscription is

$$u_2(\theta) = (1 - \alpha x_2) V_1 + \alpha x_2 V_2 - (1 - \alpha x_2) \frac{N(x_1 + x_2(1 - \alpha x_2))}{Q} \theta - p_1 - p_2.$$

We have similar insights by further considering WiFi congestion.

Operators' Profits after the Introduction of the Crowdsourced WiFi

Both x_1 and x_2 fractions of users pay the 5G operator with price p_1, the 5G operator's profit changes to:

$$\pi_1 = N(x_1 + x_2)p_1,$$

Li et al., "Optimal Pricing for Peer-to-Peer Sharing With Network Externalities," in IEEE/ACM Transactions on Networking, vol. 29, no. 1, pp. 148-161, 2021.

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The crowdsourced WiFi operator selfishly decides price p_2 to the fraction x_2 of users, and his profit is:

$$\pi_2 = Nx_2(p_2 - c),$$

c: the deployment cost per user/AP to install and add to the crowdsourced WiFi network.

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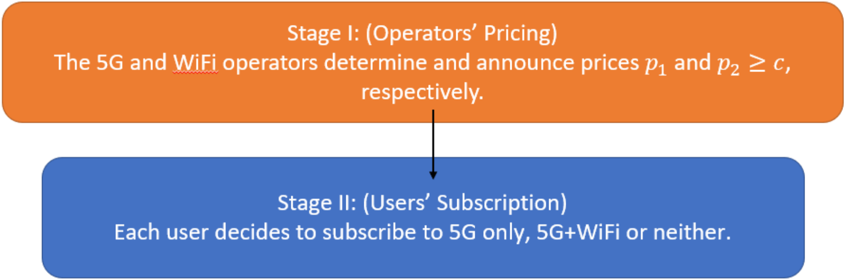
Two-stage Dynamic Game



We expect more users' subscription after the introduction of the crowdsourced WiFi.

Two-stage Dynamic Game

Stage I: (Operators' Pricing)
The 5G and WiFi operators determine and announce prices p_1 and $p_2 \geq c$, respectively.

Stage II: (Users' Subscription)
Each user decides to subscribe to 5G only, 5G+WiFi or neither.

We expect more users' subscription after the introduction of the crowdsourced WiFi.

We derive equilibrium with backward induction.

Users' Subscription in Stage II

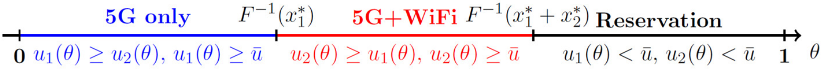


Fig. 3(a)

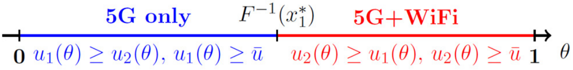


Fig. 3(b)

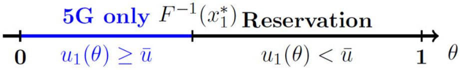
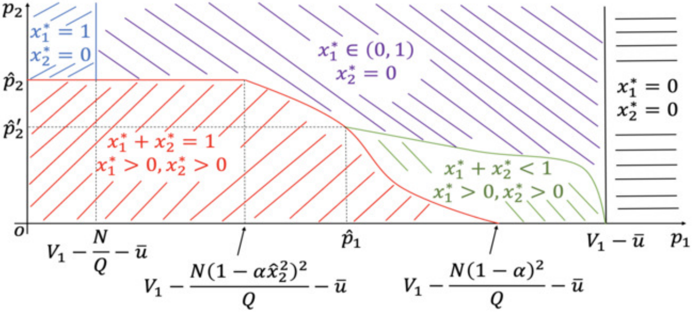


Fig. 3(c)

- Congestion-insensitive users join 5G only to avoid additional payment.
- Congestion-sensitive users may join 5G+WiFi for better experience.

Equilibrium in Stage II



If both prices of the 5G and the add-on WiFi services are low (see the lower left region), all the users either choose 5G or 5G+WiFi service.

5G Operator's Equilibrium Profit in Stage I

We prove that after the introduction of the crowdsourced WiFi, the 5G operator obtains

- at least the same profit,

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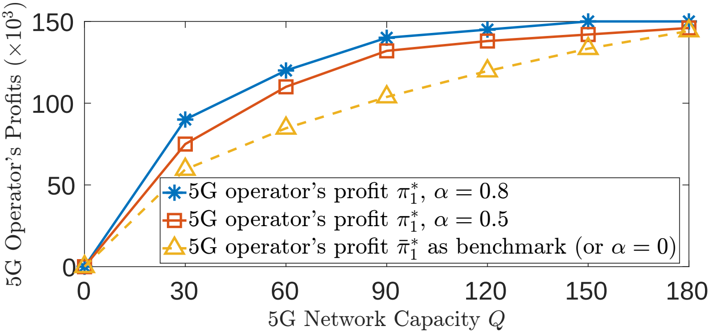

Non-small 5G capacity brings less improvement on 5G profit after the introduction of the crowdsourced WiFi.

5G Operator's Pricing & Users' Payoffs

We prove that in large regime of 5G network capacity,

- the 5G operator charges strictly greater price.
- all the users obtain strictly less payoff.

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In medium regime of 5G network capacity,

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  with small WiFi deployment cost and non-small AP coverage.

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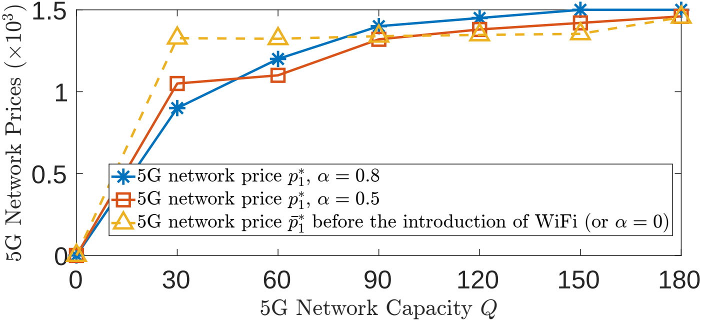
In small regime of 5G network capacity,

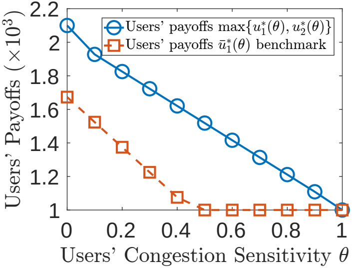
- the 5G operator charges strictly smaller price.

Numerical Results on 5G's Pricing

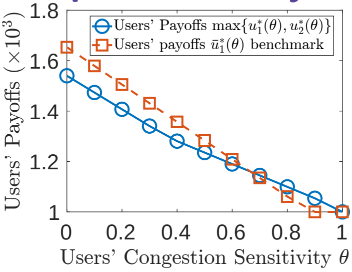


Small 5G network capacity Q=30,
decreased 5G price with increased AP coverage α.

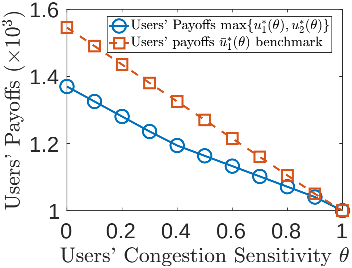
Numerical Results on Users' Equilibrium Payoffs



(c) Small Capacity $Q = 30$



(d) Small Capacity $Q = 120$



(e) Non-small capacity $Q = 180$

1. Part I: To Help or Disturb: Introduction of Crowdsourced WiFi to 5G Networks
    - Background: WiFi's Complementarity for 5G Networks
    - System Model
    - Equilibrium Analysis
    - Summary

Summary

We considered

- WiFi's complementarity for 5G networks.
- Both incur diverse network externalities.

We showed that after the introduction of the crowdsourced WiFi,

- 5G operator obtains more profit,
- 5G's structural pricing,
- all the users' payoffs may be worse.

We extended to congested WiFi with similar insights.

Part I:
To Help or Disturb: Introduction of Crowdsourced WiFi to 5G Networks

Part II:
Regulating Competition in Age of Information under Network Externalities

2 Part II: Regulating Competition in Age of Information under Network Externalities

- **Background on Economics of AoI**
- System Model for AoI
- Complete Information Scenario
- Main Results under Complete Information
- Incomplete Information Scenario
- Interesting Results under Incomplete Information

Background: Who care about AoI?

- Age of Information (AoI): duration from the moment that the latest content was generated to current reception time.

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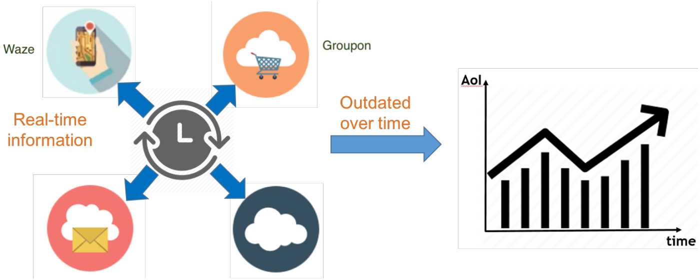
- Age of Information (AoI): duration from the moment that the latest content was generated to current reception time.
- Today many customers do not want to lose any breaking news or useful information in smartphone even if in minute.
- Online content platforms (such as navigation and shopping applications) aim to keep their information update fresh.



Background: Crowdsourcing for reducing AoI

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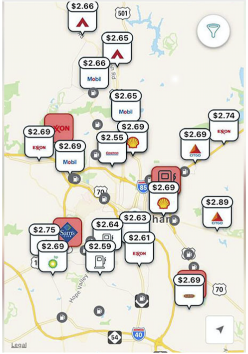
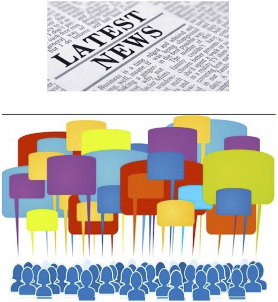


GasBuddy



CrowdSpark

Background: Sampling Cost in Crowdsourcing

Crowdsourcing platforms incur large sampling cost with high sampling rate.

Highlighted items are only available in the App.

Points available to earn every day

How Points are earned	Points per	Maximum per Day
Report / Update A Price	200	1000
Challenges	varying	varying
Read a Recent News Item	25	125
Post A Message In The Forums	100	500

Points available to earn once per week

How Points are earned	Maximum per Week
Vote in Weekly Poll	100
Weekly Challenge	varying

Points available to earn once per month

How Points are earned	Maximum per Month
Monthly Challenge	varying

Points available to earn with a Life Time Maximum

How Points are earned	Points per	Maximum Life Time
Tell-a-Friend	80	1200
Become a Member	1000	1000
Total (Daily or Weekly points not included)		**2200**

Points are all limited in some way, to help prevent abuse (for example: people reporting false prices for more points).

Figure: GasBuddy point system: payment for gas price update.

JLuo, "GasBuddy: A Platform for Crowdsourced Gas Price Data," Harvard Digital Innovation and Transformation, 2018.

Background: Sampling Cost in Crowdsourcing

Crowdsourcing platforms incur sampling cost with high sampling rate.

Huge amount of information for processing

- Waze has at least 140 millions active users.
- Gasbuddy has price information of 150,000 gas stations and 100 million downloads.

Waze, https://www.waze.com/about/.
Gasbuddy, https://www.gasbuddy.com/about/.

Research Questions

Economic Issue on AoI was largely overlooked in the literature.

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How to best tradeoff between AoI reduction and sampling cost?

How bad is platform competition and how to enforce efficient cooperation between selfish platforms?

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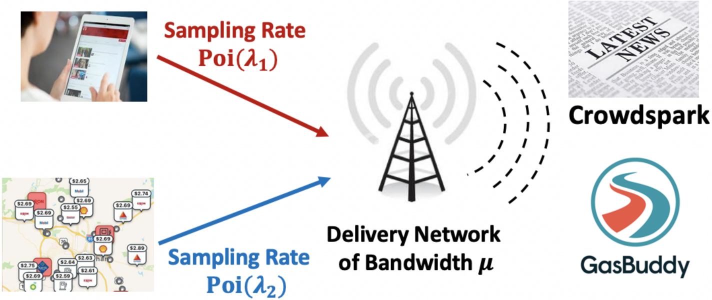
System Model on Platforms



Two platforms Crowdspark and GasBuddy need to decide how many samples to buy from their own crowdsourcing pool with sampling rates λ_1 and λ_2, and then update to their end customers through the delivery network of bandwidth μ.

Radio Spectrum Allocation, Federal Communications Commission, https://www.fcc.gov/engineering-technology/policy-and-rules-division/general/radio-spectrum-allocation#:~:text=Currently%20only%20frequency%20bands%20between,astronomy%20service%20under%20specified%20conditions).

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We consider LCFS M/M/1 queue with preemption.

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Negative network externalities due to competition on μ.

System Model under Complete Information

Model c_i as unit cost per sampling rate. Sampling cost is $\lambda_i c_i$ when inviting sensors of density λ_i to contribute.

Platform 1

Platform 2

GasBuddy

c_1 →

← c_2

Sampling rate: λ_1
Sampling cost: c_1 (per sampling rate)

Sampling rate: λ_2
Sampling cost: c_2 (per sampling rate)

Both platforms have full information on their sampling costs.

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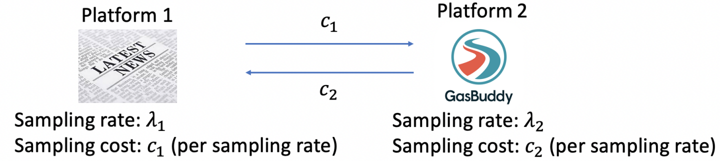

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- Platform 1's cost function:

$$\pi_1(\lambda_1, \lambda_2) = \Delta_1(\lambda_1, \lambda_2) + c_1\lambda_1.$$

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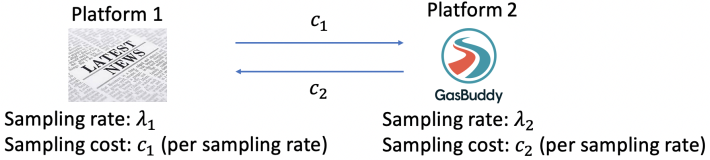

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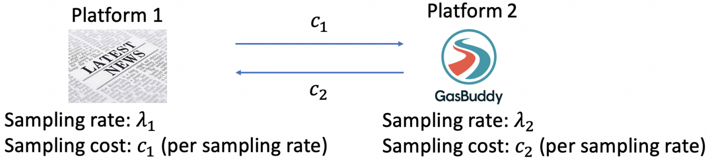

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- Social cost function:

$$\pi(\lambda_1, \lambda_2) = \pi_1(\lambda_1, \lambda_2) + \pi_2(\lambda_1, \lambda_2).$$

Non-cooperative Static Game under Complete Information

- Non-cooperative game with equilibrium $(\lambda_1^*, \lambda_2^*)$

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- Price of Anarchy (PoA):

$$PoA = \max_{c_1, c_2, \mu} \frac{\pi(\lambda_1^*, \lambda_2^*)}{\pi(\lambda_1^{**}, \lambda_2^{**})} \geq 1.$$

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- $PoA = \infty$ in the worst case.

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Definition 1 (Non-forgiving trigger mechanism of punishment under complete information)

- *In each round, recommended cooperation profile $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ to follow, if neither was detected to deviate from its profile in the past.*
- *Once a deviation was found in the past, the two platforms will keep playing the punishment/equilibrium profile $(\lambda_1^*, \lambda_2^*)$ forever.*

Condition for No Deviation for Both Platforms

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- We assume $c_1 \leq c_2$. Which platform is more likely to deviate?
- Platform 1 is more likely to oversample and deviate with $\delta_{th_1} \geq \delta_{th_2}$.

Cooperation Profile for Large δ Regime

Large δ Regime: $\delta \geq \max\{\delta_{th_1}, \delta_{th_2}\} = \delta_{th_1}$.

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Proposition 3 (Large δ Regime)

Under complete information, if $\delta \geq \delta_{th_1}$, both platforms will follow the perfect cooperation profile $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta)) = (\lambda_1^{**}, \lambda_2^{**})$ all the time without triggering the punishment profile $(\lambda_1^*, \lambda_2^*)$.

Cooperation Profile Design for Medium δ Regime

- $\delta < \max\{\delta_{th_1}, \delta_{th_2}\}$, we cannot use $(\lambda_1^{**}, \lambda_2^{**})$ as cooperation profile.

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- $\tilde{\lambda}_1(\delta) < \lambda_1^{*}$: under-sample than equilibrium.
- $\tilde{\lambda}_1(\delta)$ decreases with $\delta \in [\delta_{th_2}, \delta_{th_1})$ and eventually $\tilde{\lambda}_1(\delta) \to \lambda_1^{**}$: platform 1 cares more about future and samples more conservative.

Cooperation Profile Design for Small δ Regime

What if $\delta < \min\{\delta_{th_1}, \delta_{th_2}\} = \delta_{th_2}$?

Cooperation Profile Design for Small δ Regime

What if $\delta < \min\{\delta_{th_1}, \delta_{th_2}\} = \delta_{th_2}$?

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- We redesign $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ jointly such that $\Pi_1(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta)) = \hat{\Pi}_1(\lambda_1^*, \lambda_2^*)$ for platform 1 and $\Pi_2(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta)) = \hat{\Pi}_2(\lambda_1^*, \lambda_2^*)$ for platform 2.

Cooperation Profile Design for Small δ Regime

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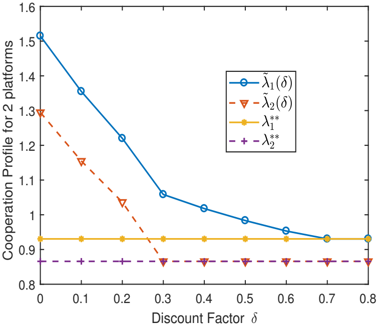
- As $\delta \to 0$, the proposed $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ approach $(\lambda_1^*, \lambda_2^*)$, and the repeated game degenerates to one-shot static game.

Numerical Results



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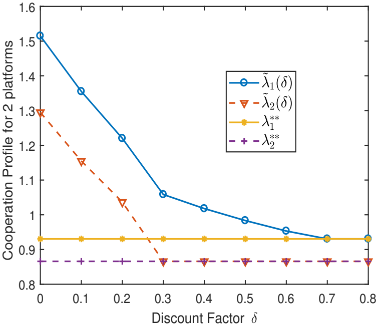

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- Cooperation profile $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ decrease with δ and converge to social optimizers $(\lambda_1^{**}, \lambda_2^{**})$.

System model under one-sided incomplete information

Platform 1

$p_H, 1 - p_H$ →

← c_2

Platform 2

GasBuddy

Sampling rate: λ_1
Sampling cost:
c_H(per sampling rate) with prob. p_H
c_L (per sampling rate) with prob. $1 - p_H$

Sampling rate: λ_2
Sampling cost: c_2 (per sampling rate)

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Bayesian game:

- Platform 1's cost function when $c_1 = c_H$:

$$\pi_1(\lambda_1(c_H), \lambda_2) = \frac{\lambda_1(c_H) + \lambda_2}{\lambda_1(c_H)} \left(\frac{1}{\lambda_1(c_H) + \lambda_2} + \frac{1}{\mu} \right) + c_H \lambda_1(c_H).$$

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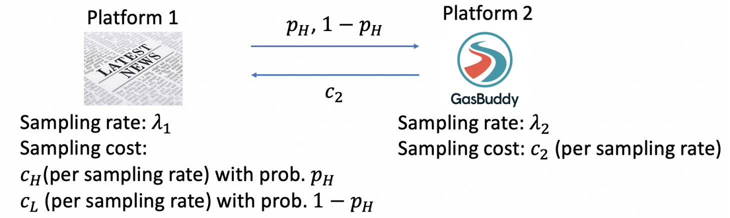

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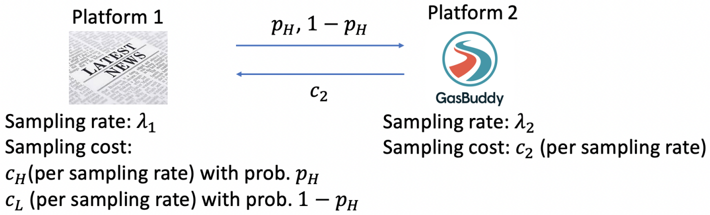

- Unaware of c_H and c_L instances, platform 2's cost function:

$$\pi_2((\lambda_1(c_H), \lambda_1(c_L)), \lambda_2) = p_H \cdot \left(\frac{\lambda_1(c_H) + \lambda_2}{\lambda_2} \left(\frac{1}{\lambda_1(c_H) + \lambda_2} + \frac{1}{\mu} \right) \right)$$
$$+ (1 - p_H) \cdot \left(\frac{\lambda_1(c_L) + \lambda_2}{\lambda_2} \left(\frac{1}{\lambda_1(c_L) + \lambda_2} + \frac{1}{\mu} \right) \right) + c_2 \lambda_2.$$

Non-cooperative Bayesian Game under Incomplete Information

- Non-cooperative Bayesian game with equilibrium $((\lambda_1^*(c_H), \lambda_1^*(c_L)), \lambda_2^*)$

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- $PoA = \infty$ in the worst case.

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Q: Does platform 1 always take advantage from knowing more information than platform 2?

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- When p_H is large, platform 1 is forced to over-sample more often and platform 1 loses in average sense.

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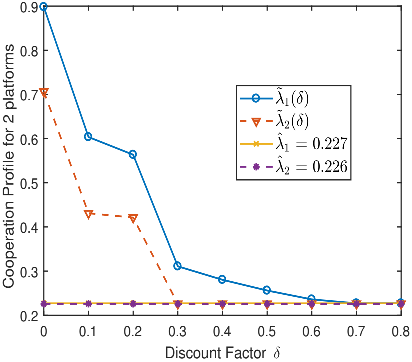
- Derive δ_{th_1} and δ_{th_2} similarly as under complete information.
- Divide profile design into three different δ regimes (low, medium and high).

Numerical Results



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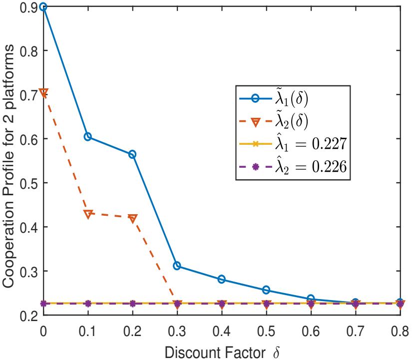

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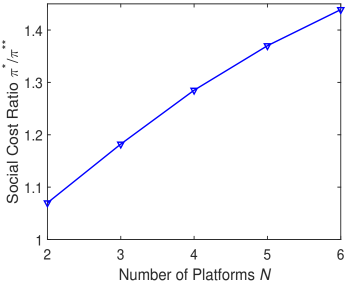
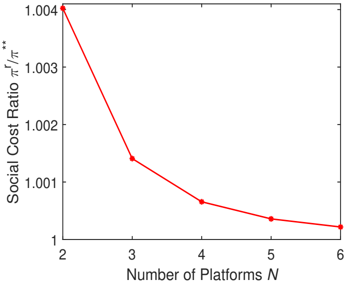
New Results with Multiple Platforms



((a)) Social cost ratio between equilibrium and optimum



((b)) Social cost ratio between approximate mechanism and optimum

Figure: Empirical performance comparison between competition equilibrium, social optimum, and our approximate mechanism here.

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- Under complete information, propose repeated games mechanism with the threat of future punishment to enforce efficient cooperation under any discount factor.
- Under incomplete information, propose approximate mechanism to negate the platform with information advantage.

Thank You! Q & A