Economics of Age of Information (AoI) Management under Network Externalities

Presenter: Shugang Hao

Pillar of Engineering Systems and Design
Singapore University of Technology and Design (SUTD)

August 31th, 2019

About SUTD

- A new public university with 10-year age.
- Was established in collaboration with MIT.
- Ranking in the world: 19th in Telecommunication Engineering according to ShanghaiRanking 2019.

Acknowledgement

- This is a joint work with Associate Professor Lingjie Duan.
- Parts of results here have appeared in ACM MobiHoc 2019.
- S. Hao and L. Duan, "Economics of Age of Information Management under Network Externalities," in *the Twentieth International Symposium on Mobile Ad Hoc Networking and Computing (ACM MobiHoc)*, 2019.

Overview

1. Background: crowdsourcing meets AoI
2. System Model for AoI
3. Complete information scenario
4. Incomplete information scenario

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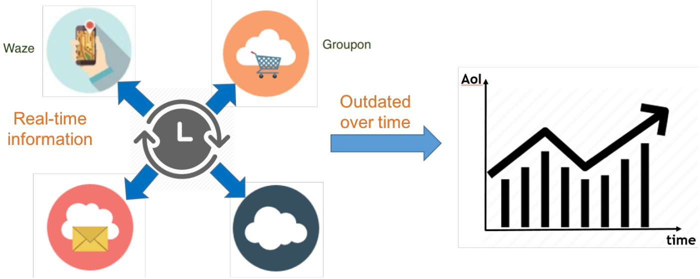
- Today many customers do not want to lose any breaking news or useful information in smartphone even if in minute.
- Online content platforms (such as navigation and shopping applications) aim to keep their information update fresh.



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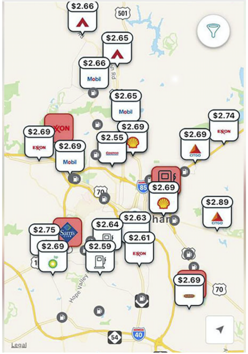
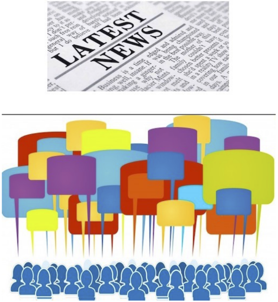


GasBuddy



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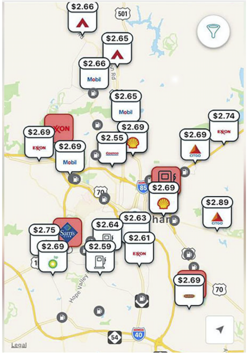

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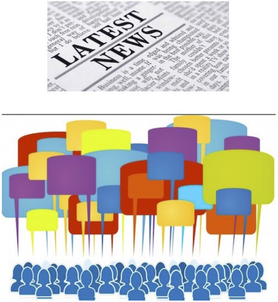

CrowdSpark

Both incur large sampling cost with high sampling rate.

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  - Updates of platforms may preempt or jam each other.
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How bad is platform competition and how to enforce efficient cooperation between selfish platforms?

Related Work on AoI

Queueing analysis on average AoI estimation

- Single link: Costa et al. (2016), Huang et al. (2015), Kaul et al. (2012), Bacinoglu et al. (2015) and Sun et al. (2017).
- Multi-hop networks: Bedewy et al. (2017).
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Scheduling broadcast channel among multiple sources for AoI to avoid competition

- Hsu et al. (2017). Bedewy et al. (2017).
- Such work assumes sources/platforms will follow recommendation, and do not consider selfish sources' update competition over the content delivery network.

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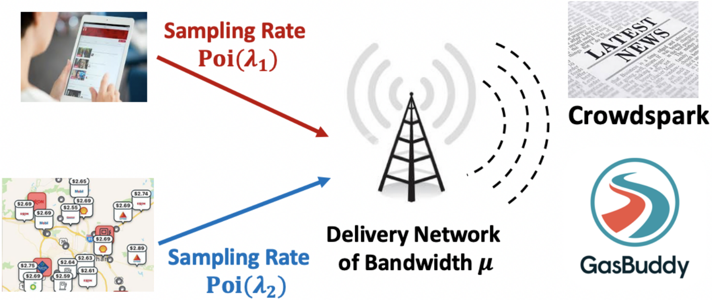
We will

- investigate how to regulate platform competition under incomplete information.
- propose non-monetary approach to work for any discount factor.

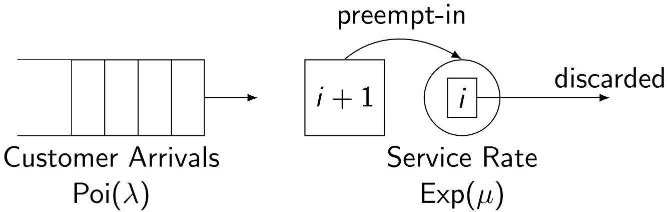
System Model on Platforms



Two platforms Crowdspark and GasBuddy need to decide how many samples to buy from their own crowdsourcing pool with sampling rates λ_1 and λ_2, and then update to their end customers through the delivery network of bandwidth μ.

System Model on AoI

Consider 1 platform first



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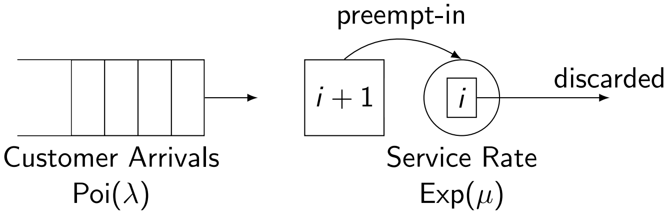

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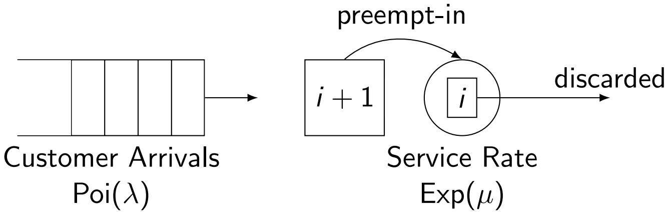

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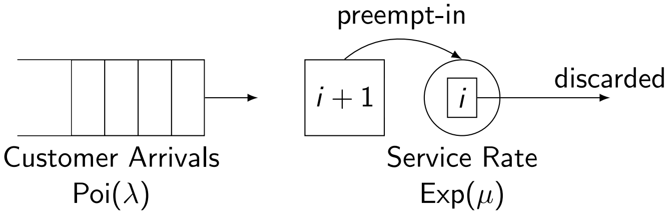

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- Preemption happen within (and between) platform(s).
- Status update age = completion time - generation time.
- Average age for single platform:

$$\Delta = \frac{1}{\lambda} + \frac{1}{\mu}.$$

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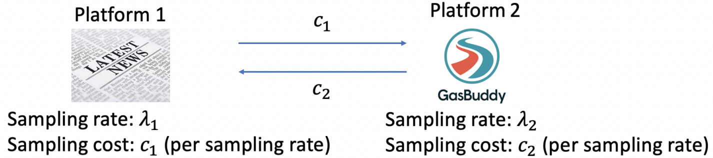
System Model under Complete Information

Model c_i as unit cost per sampling rate. Sampling cost is $\lambda_i c_i$ when inviting sensors of density λ_i to contribute.



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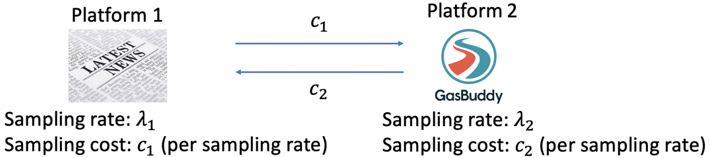

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- Platform 1's cost function:

$$\pi_1(\lambda_1, \lambda_2) = \Delta_1(\lambda_1, \lambda_2) + c_1\lambda_1.$$

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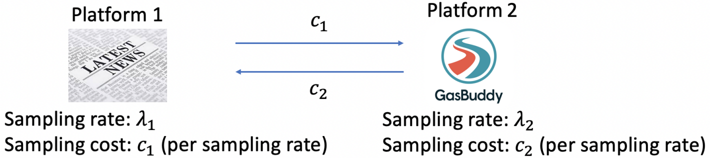

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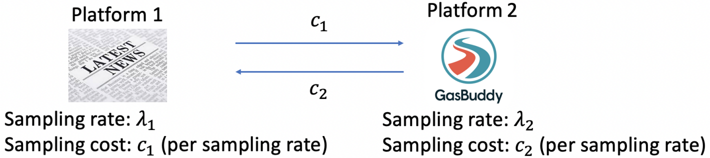

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- Social cost function:

$$\pi(\lambda_1, \lambda_2) = \pi_1(\lambda_1, \lambda_2) + \pi_2(\lambda_1, \lambda_2).$$

Non-cooperative Static Game under Complete Information

- Non-cooperative game with equilibrium $(\lambda_1^*, \lambda_2^*)$

$$\min_{\lambda_1 > 0} \pi_1(\lambda_1, \lambda_2)$$
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Question: equilibrium $(\lambda_1^*, \lambda_2^*)$ versus optimal $(\lambda_1^{**}, \lambda_2^{**})$?

Competition Equilibrium and Social Optimizers

Proposition 1 (Equilibrium vs Social Optimizers under complete information)

Under complete information, the competition equilibrium $(\lambda_1^*, \lambda_2^*)$ are the unique solutions to

$$\begin{aligned} -\frac{1}{\lambda_1^2}(1+\frac{\lambda_2}{\mu}) + c_1 = 0, \\ -\frac{1}{\lambda_2^2}(1+\frac{\lambda_1}{\mu}) + c_2 = 0. \end{aligned} \tag{1}$$

Differently, the social optimizers $(\lambda_1^{**}, \lambda_2^{**})$, are the unique solutions to

$$\begin{aligned} -\frac{1}{\lambda_1^2}(1+\frac{\lambda_2}{\mu}) + c_1 + \frac{1}{\lambda_2 \mu} = 0, \\ -\frac{1}{\lambda_2^2}(1+\frac{\lambda_1}{\mu}) + c_2 + \frac{1}{\lambda_1 \mu} = 0. \end{aligned} \tag{2}$$

By comparing (1) and (2), we conclude competition leads over-sampling ($\lambda_i^* \geq \lambda_i^{**}$ for $i = 1, 2$) at the equilibrium.

Equilibrium under Complete Informatiion

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- λ_1^* decreases with c_2: λ_2 decreases with c_2.

Inefficiency of Competition Equilibrium under Complete Information

Price of Anarchy (PoA):

$$PoA = \max_{c_1, c_2, \mu} \frac{\pi(\lambda_1^*, \lambda_2^*)}{\pi(\lambda_1^{**}, \lambda_2^{**})} \geq 1.$$

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Definition 1 (Non-forgiving trigger mechanism of punishment under complete information)

- *In each round, recommended cooperation profile* $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ *to follow, if neither was detected to deviate from its profile in the past.*
- *Once a deviation was found in the past, the two platforms will keep playing the punishment/equilibrium profile* $(\lambda_1^*, \lambda_2^*)$ *forever.*

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$$\Pi_1 = \pi_1(\lambda_1^{**}, \lambda_2^{**}) + \delta\pi_1(\lambda_1^{**}, \lambda_2^{**}) + \delta^2\pi_1(\lambda_1^{**}, \lambda_2^{**}) + \cdots .$$

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- Platform 1's long-term cost over all time stages by deviating in the first round with best response $\lambda_1 = \sqrt{\frac{1+\lambda_2^{**}/\mu}{c_1}}$:

$$\hat{\Pi}_1 = \pi_1\left(\sqrt{\frac{1+\lambda_2^{**}/\mu}{c_1}}, \lambda_2^{**}\right) + \underbrace{\delta \pi_1(\lambda_1^{*}, \lambda_2^{*}) + \delta^2 \pi_1(\lambda_1^{*}, \lambda_2^{*}) + \cdots}_{\text{Equilibrium as punishment}} .$$

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$$\text{discount factor } \delta \geq \delta_{th_1} := \frac{\left(\sqrt{\frac{1+\lambda_2^{**}/\mu}{c_1+\frac{1}{\lambda_2^{**}\mu}}} - \sqrt{\frac{1+\lambda_2^{**}/\mu}{c_1}}\right)^2}{2\lambda_1^{**}\left(\lambda_1^* - \sqrt{\frac{1+\lambda_2^{**}/\mu}{c_1}}\right)}.$$

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- Platform 1 is more likely to oversample and deviate with $\delta_{th_1} \geq \delta_{th_2}$.

Cooperation Profile for Large δ Regime

Large δ Regime: $\delta \geq \max\{\delta_{th_1}, \delta_{th_2}\} = \delta_{th_1}$.

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Proposition 3 (Large δ Regime)

Under complete information, if $\delta \geq \delta_{th_1}$, both platforms will follow the perfect cooperation profile $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta)) = (\lambda_1^{**}, \lambda_2^{**})$ all the time without triggering the punishment profile $(\lambda_1^{*}, \lambda_2^{*})$.

Cooperation Profile Design for Medium δ Regime

- $\delta < \max\{\delta_{th_1}, \delta_{th_2}\}$, we cannot use $(\lambda_1^{**}, \lambda_2^{**})$ as cooperation profile.

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Proposition 4 (Medium δ Regime)

If $\delta_{th_2} \leq \delta < \delta_{th_1}$, cooperation profile for platform 1 $\tilde{\lambda}_1(\delta)$ satisfies:

- $\tilde{\lambda}_1(\delta) > \lambda_1^{**}$: over-sample than social optimizer.

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- $\tilde{\lambda}_1(\delta) > \lambda_1^{**}$: over-sample than social optimizer.
- $\tilde{\lambda}_1(\delta) < \lambda_1^{*}$: under-sample than equilibrium.
- $\tilde{\lambda}_1(\delta)$ decreases with $\delta \in [\delta_{th_2}, \delta_{th_1})$ and eventually $\tilde{\lambda}_1(\delta) \to \lambda_1^{**}$: platform 1 cares more about future and samples more conservative.

Cooperation Profile Design for Small δ Regime

What if $\delta < \min\{\delta_{th_1}, \delta_{th_2}\} = \delta_{th_2}$?

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- We redesign $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ jointly such that $\Pi_1(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta)) = \hat{\Pi}_1(\lambda_1^*, \lambda_2^*)$ for platform 1 and $\Pi_2(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta)) = \hat{\Pi}_2(\lambda_1^*, \lambda_2^*)$ for platform 2.

Cooperation Profile Design for Small δ Regime

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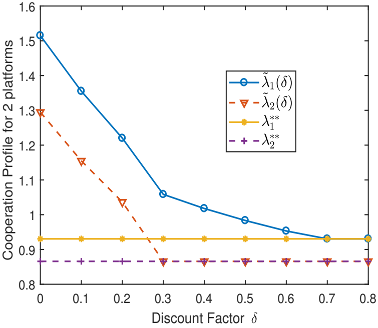
- As $\delta \to 0$, the proposed $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ approach $(\lambda_1^*, \lambda_2^*)$, and the repeated game degenerates to one-shot static game.

Numerical Results



Low δ regime: 0 - 0.3, Medium δ regime: 0.3 - 0.7, High δ regime: 0.7 - 1.

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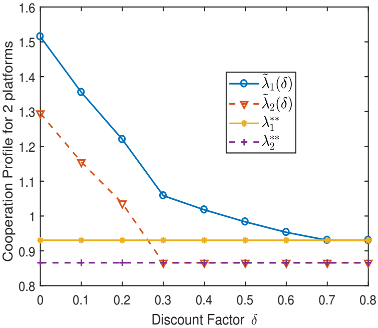

Low δ regime: 0 - 0.3, Medium δ regime: 0.3 - 0.7, High δ regime: 0.7 - 1.

- Cooperation profile $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ decrease with δ and converge to social optimizers $(\lambda_1^{**}, \lambda_2^{**})$.

System model under one-sided incomplete information

Platform 1

$p_H, 1 - p_H$

c_2

Platform 2

GasBuddy

Sampling rate: λ_1
Sampling cost:
c_H(per sampling rate) with prob. p_H
c_L (per sampling rate) with prob. $1 - p_H$

Sampling rate: λ_2
Sampling cost: c_2 (per sampling rate)

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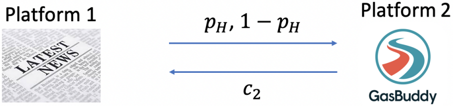

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Sampling cost:
c_H(per sampling rate) with prob. p_H
c_L (per sampling rate) with prob. $1-p_H$

Sampling rate: λ_2
Sampling cost: c_2 (per sampling rate)

Bayesian game:

- Platform 1's cost function when $c_1 = c_H$:

$$\pi_1(\lambda_1(c_H), \lambda_2) = \frac{\lambda_1(c_H)+\lambda_2}{\lambda_1(c_H)}\left(\frac{1}{\lambda_1(c_H)+\lambda_2}+\frac{1}{\mu}\right)+c_H\lambda_1(c_H).$$

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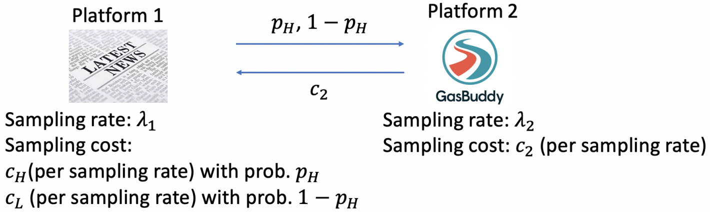

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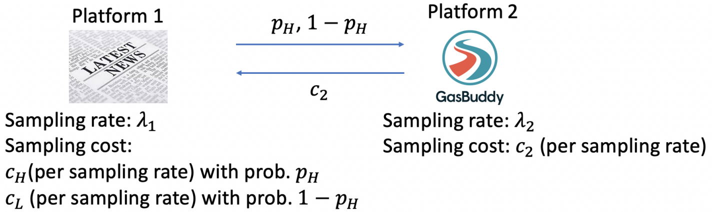
- Platform 1's cost function when $c_1 = c_L$:

$$\pi_1(\lambda_1(c_L), \lambda_2) = \frac{\lambda_1(c_L)+\lambda_2}{\lambda_1(c_L)}\left(\frac{1}{\lambda_1(c_L)+\lambda_2}+\frac{1}{\mu}\right) + c_L\lambda_1(c_L).$$

System model under one-sided incomplete information



- Unaware of c_H and c_L instances, platform 2's cost function:

$$\pi_2((\lambda_1(c_H), \lambda_1(c_L)), \lambda_2) = p_H \cdot \left(\frac{\lambda_1(c_H) + \lambda_2}{\lambda_2} \left(\frac{1}{\lambda_1(c_H) + \lambda_2} + \frac{1}{\mu} \right) \right)$$

$$+ (1 - p_H) \cdot \left(\frac{\lambda_1(c_L) + \lambda_2}{\lambda_2} \left(\frac{1}{\lambda_1(c_L) + \lambda_2} + \frac{1}{\mu} \right) \right) + c_2 \lambda_2.$$

Non-cooperative Bayesian Game under Incomplete Information

- Non-cooperative Bayesian game with equilibrium $((\lambda_1^*(c_H), \lambda_1^*(c_L)), \lambda_2^*)$:

$$\min_{\lambda_1(c_H)>0} \pi_1(\lambda_1(c_H), \lambda_2)$$

$$\min_{\lambda_1(c_L)>0} \pi_1(\lambda_1(c_L), \lambda_2)$$

$$\min_{\lambda_2>0} \pi_2((\lambda_1(c_H), \lambda_1(c_L)), \lambda_2)$$

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$$\min_{\lambda_2>0} \pi_2((\lambda_1(c_H), \lambda_1(c_L)), \lambda_2)$$

- Min-social-cost problem with social optimizers $((\lambda_1^{**}(c_H), \lambda_1^{**}(c_L)), \lambda_2^{**})$:

$$\min_{\lambda_1(c_H),\lambda_1(c_L),\lambda_2>0} \pi((\lambda_1(c_H), \lambda_1(c_L)), \lambda_2)$$

Competition Equilibrium and Social Optimizers

Proposition 6 (Equilibrium vs social optimizers under incomplete information)

The competition equilibrium $((\lambda_1^*(c_H), \lambda_1^*(c_L)), \lambda_2^*)$ are the unique solutions to

$$-\frac{1}{\lambda_1^2(c_H)}\left(1+\frac{\lambda_2}{\mu}\right)+c_H=0,$$
$$-\frac{1}{\lambda_1^2(c_L)}\left(1+\frac{\lambda_2}{\mu}\right)+c_L=0,$$
$$-\frac{p_H}{\lambda_2^2}\left(1+\frac{\lambda_1(c_H)}{\mu}\right)-\frac{1-p_H}{\lambda_2^2}\left(1+\frac{\lambda_1(c_L)}{\mu}\right)+c_2=0.$$

Social optimizers $((\lambda_1^{**}(c_H), \lambda_1^{**}(c_L)), \lambda_2^{**})$ are the unique solutions to

$$-\frac{1}{\lambda_1^2(c_H)}\left(1+\frac{\lambda_2}{\mu}\right)+c_H+\frac{1}{\lambda_2\mu}=0,$$
$$-\frac{1}{\lambda_1^2(c_L)}\left(1+\frac{\lambda_2}{\mu}\right)+c_L+\frac{1}{\lambda_2\mu}=0,$$
$$p_H\left(-\frac{1}{\lambda_2^2}\left(1+\frac{\lambda_1(c_H)}{\mu}\right)+c_2+\frac{1}{\lambda_1(c_H)\mu}\right)+(1-p_H)\left(-\frac{1}{\lambda_2^2}\left(1+\frac{\lambda_1(c_L)}{\mu}\right)+c_2+\frac{1}{\lambda_1(c_L)\mu}\right)=0.$$

Both platforms will over-sample at equilibrium, i.e., $\lambda_1^*(c_H)\geq\lambda_1^{**}(c_H)$, $\lambda_1^*(c_L)\geq\lambda_1^{**}(c_L)$ and $\lambda_2^*\geq\lambda_2^{**}$. Additionally, $\lambda_1^*(c_H)/\lambda_1^*(c_L)=\sqrt{c_L/c_H}$.

Inefficiency of Competition Equilibrium

Price of Anarchy (PoA):

$$PoA = \max_{c_L, c_H, c_2, \mu, p_H} \frac{\pi((\lambda_1^*(c_H), \lambda_1^*(c_L)), \lambda_2^*)}{\pi((\lambda_1^{**}(c_H), \lambda_1^{**}(c_L)), \lambda_2^{**})} \geq 1.$$

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Price of anarchy under incomplete information is $PoA = \infty$, which is achieved when platform 1's smaller sampling cost c_L is infinitesimal.

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Need non-monetary mechanism to remedy huge efficiency loss!

Hurt with More Information for Platform 1

Question: Does platform 1 take advantage from knowing more information about the sampling costs of both platforms?

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Answer: Not exactly even in average sense!

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Under incomplete information, the cost objective of platform 1 under each $c_1 = c_H$ realization is greater than that under complete information, and becomes smaller under each $c_1 = c_L$ realization.

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Under incomplete information, the cost objective of platform 1 under each $c_1 = c_H$ realization is greater than that under complete information, and becomes smaller under each $c_1 = c_L$ realization.

Perhaps surprisingly, its average cost $p_H \pi_1(\lambda_1^*(c_H), \lambda_2^*) + (1 - p_H)\pi_1(\lambda_1^*(c_L), \lambda_2^*)$ becomes greater once

$$p_H \geq \frac{\sqrt{c_L}(\sqrt{1 + \bar{\lambda}_2(c_L)/\mu} - \sqrt{1 + \lambda_2^*/\mu})}{\sqrt{c_L}(\sqrt{1 + \bar{\lambda}_2(c_L)/\mu} - \sqrt{1 + \lambda_2^*/\mu}) + \sqrt{c_H}(\sqrt{1 + \lambda_2^*/\mu} - \sqrt{1 + \bar{\lambda}_2(c_H)/\mu})}$$

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- Platform 2 cannot identify $c_1 = c_H$ or $c_1 = c_L$, and its over-sampling when $c_1 = c_H$ forces platform 1 to over-sample.

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- When p_H is large, this happens more often and platform 1 loses in average sense.

Approximate Mechanism under Incomplete Information

New Challenge for Mechanism Design under Incomplete Information

Even if δ is large enough, can we still use social optimizers $((\lambda_1^{**}(c_L), \lambda_1^{**}(c_H)), \lambda_2^{**})$ as in complete information scenario?

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Lemma 5.1.

Given the cooperation profile $(\lambda_1^{**}(c_L), \lambda_1^{**}(c_H))$ for platform 1 under sufficiently large δ, platform 1 will not deviate from $\lambda_1^{**}(c_L)$ when $c_1 = c_L$ but may deviate from $\lambda_1^{**}(c_H)$ when $c_1 = c_H$.

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- Platform 2 over-samples when $c_1 = c_H$ compared to complete information scenario.
- Platform 1 benefits from choosing $\lambda_1^{**}(c_L)$ and sampling more than $\lambda_1^{**}(c_H)$.
- When sampling according to larger $\lambda_1^{**}(c_L)$, he will not be caught.

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- Platform 2 over-samples when $c_1 = c_H$ compared to complete information scenario.
- Platform 1 benefits from choosing $\lambda_1^{**}(c_L)$ and sampling more than $\lambda_1^{**}(c_H)$.
- When sampling according to larger $\lambda_1^{**}(c_L)$, he will not be caught.

Need to design new $((\tilde{\lambda}_1(c_H, \delta), \tilde{\lambda}_1(c_L, \delta)), \tilde{\lambda}_2(\delta))$ even for large δ regime!

Approximate Profile Design under Incomplete Information

New idea: recommend platform 1 to behave indifferently no matter $c_1 = c_H$ or $c_1 = c_L$. That is, $\tilde{\lambda}_1(c_H, \delta) = \tilde{\lambda}_1(c_L, \delta) = \tilde{\lambda}_1(\delta)$.

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Definition 2 (Approximate trigger mechanism of punishment under incomplete information)

- *In each round, two platforms follow approximate cooperation profile* $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ *if neither was detected to deviate from tits profile in the past.*
- *Once a deviation was found in the past, the two platforms will keep playing the equilibrium punishment profile* $((\lambda_1^*(c_H), \lambda_1^*(c_L)), \lambda_2^*)$ *forever.*

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What is the best design for $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$?

Approximate Cooperation Profile

New min-social cost problem with optimal profile $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$:

$$\min_{\lambda_1(c_H,\delta),\lambda_1(c_L,\delta),\lambda_2(\delta)>0} \pi((\lambda_1(c_H,\delta), \lambda_1(c_L,\delta)), \lambda_2(\delta))$$

$$s.t. \quad \lambda_1(c_H,\delta) = \lambda_1(c_L,\delta) := \tilde{\lambda}_1(\delta)$$

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Optimal approximate profile:

$$\tilde{\lambda}_1(\delta)=\sqrt{\frac{1+\tilde{\lambda}_2(\delta)/\mu}{p_H c_H+(1-p_H)c_L+\frac{1}{\tilde{\lambda}_2(\delta)\mu}}},$$
$$\tilde{\lambda}_2(\delta)=\sqrt{\frac{1+\tilde{\lambda}_1(\delta)/\mu}{c_2+\frac{1}{\tilde{\lambda}_1(\delta)\mu}}},$$

which proves to provide at most 2-approximation of minimum social cost with $p_H c_H+(1-p_H)c_L=c_2$.

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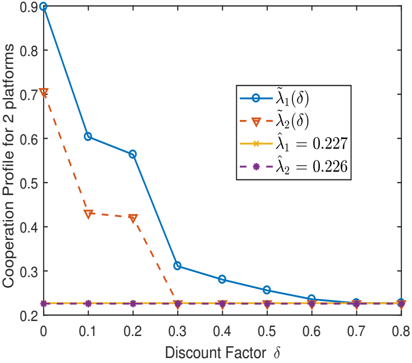
- Derive δ_{th_1} and δ_{th_2} similarly as under complete information.

Approximate Mechanism Design under Incomplete Information

- Derive δ_{th_1} and δ_{th_2} similarly as under complete information.
- Divide profile design into three different δ regimes (low, medium and high):
  - High δ regime ($\delta \geq \delta_{th1}$): both platforms follow optimal recommendation.
  - Medium δ regime ($\delta_{th2} \leq \delta < \delta_{th1}$): only one platform follows optimal recommendation.
  - Low δ regime ($\delta < \delta_{th2}$): neither follows optimal recommendation.

Numerical Results



Low δ regime: 0 - 0.3, Medium δ regime: 0.3 - 0.7, High δ regime: 0.7 - 1

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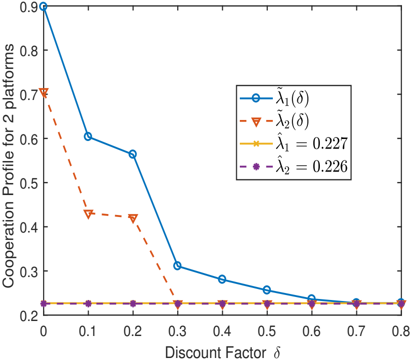

Low δ regime: 0 - 0.3, Medium δ regime: 0.3 - 0.7, High δ regime: 0.7 - 1

- Cooperation profile $(\tilde{\lambda}_1(\delta), \tilde{\lambda}_2(\delta))$ decrease with δ and converge to optimal recommended profile.

Conclusion

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- Competition between platforms when co-using content delivery network can lead to huge efficiency loss ($PoA \to \infty$) under both complete and incomplete info.
- Under complete information, propose repeated games mechanism with the threat of future punishment to enforce efficient cooperation under any discount factor.
- Under incomplete information, propose approximate mechanism to negate the platform with information advantage.

Extensions on JSAC Submission

- Multi-platform scenario under complete information.

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- At most $\frac{N}{N-1}$ of minimum social cost given symmetric costs under incomplete information.

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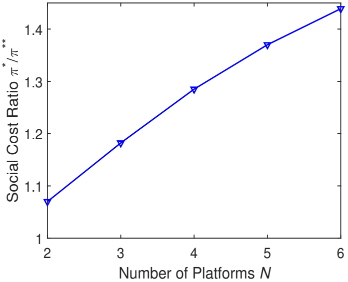
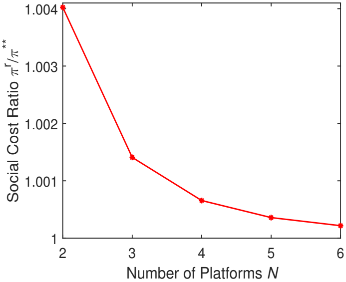


((a)) Social cost ratio between equilibrium and optimum



((b)) Social cost ratio between approximate mechanism and optimum

Figure: Empirical performance comparison between competition equilibrium, social optimum, and our approximate mechanism here.

Thank You! Q & A